Naturally Light Neutrinos and Unification in Theories with Low Scale Quantum Gravity

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Abstract

Within low scale theories traditional see-saw and scalar triplet mechanisms, for neutrino mass suppression, do not work out anymore and for realistic model building some new ideas are needed. In this paper we suggest mechanism, different from existing ones, which provides natural suppression of the neutrino masses. The mechanism is realized through extended scalars of 4, 5 or 6 dimensional $SU(2)_L$ multiplets. Scenario, with fundamental mass scale M_f in a $\sim 10^3$ TeV range, requires 4-plets guaranteeing neutrino masses $\lesssim 1$ eV. For theories with $M_f = $ few $\cdot$ 10 TeV 5-plets should be involved, while in scenarios with $M_f = $ few TeV, 6-plets could be efficient.

The considered mechanism could be successfully applied also for supersymmetric theories, building scenarios with various values of low M_f.

Within considered models we also address the question of gauge coupling unification. For low scale unification, existence of compact extra dimensions turns out to be crucial. Due to additional scalar multiplets, some new examples of unification are found for both - non SUSY and SUSY cases. Within non SUSY scenarios introduced $SU(2)_L$ scalars take advantage and are important for successful unification.

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1 Introduction

Theories with extra spacetime dimensions have attracted great attention for last years. Main phenomenological motivation, for considering such type of scenarios, was the new possibilities of resolution of gauge hierarchy problem [1]-[3]. It was observed [1] that due to appropriately large extra dimensions, it is possible to lower fundamental scale M_f even down to few TeV, while the observed weakness of gravity could be explained through large volume of extra space. In fact, for extra spacelike dimension's number $\delta = 2$, four dimensional Planck mass M_{Pl} has value $\sim 10^{19}$ GeV if size R of extra compact dimensions is in a range $\lesssim 1$ mm (distance at which the behavior of gravity is still unknown and is extensively investigated in upgoing experiments [4]).

An alternative approach has been suggested in Refs. [2, 3]. In these scenarios, although the fundamental scale can be close to M_{Pl}, the required hierarchy is obtained on a 'visible' brane through non-factorizable geometry. Latter solution emerges from higher dimensional gravity with one [2, 3] or more [5] extra dimensions.

In both type, of above mentioned scenarios, it is assumed that all Standard Model (SM) fields are confined to a 3-brane (identified with our Universe) in extra dimensions. The idea, that we live on a brane/topological defect embedded in a higher dimensional space, goes back to [6].

Despite great success in solving the gauge hierarchy problem, within this type of theories, various problems arise and numerous issues require to be reconsidered from a new viewpoint. This cast an intriguing challenge to theoreticians. Amongst raised issues, the actual task is to understand how to suppress neutrino masses in a needed level. Due to low fundamental scale, the well known see-saw [7] and scalar triplet [8] mechanisms do not lead to the sufficient suppressions. Of course, by requiring conservation of lepton number, it is possible to restrict operators responsible for neutrino masses. On the other hand, latest atmospheric [9] and solar [10] neutrino data have increased confidence in the neutrino oscillations. So, the purpose is to generate neutrino masses with desirable magnitudes. In [11] there were suggested mechanisms, which have extra dimensional nature and successfully resolve this problem. One way is to couple bulk right handed neutrinos with left handed ones. In this case suppression occurs in Dirac Yukawa couplings through the large volume factor [11, 12]. Needed suppression also can be achieved if lepton number violation takes place on a distant brane [11]. Different approach was presented in [13], where for suppression of neutrino masses, together with right handed neutrinos was introduced additional scalar doublet with a sufficiently tiny VEV.

In this paper we suggest mechanism, which provides generation of suppressed neutrino masses. The mechanism do not has extra dimensional nature. Suppression occurs due to group-theoretical reason. Namely, by introducing extended charged $SU(2)_L$ scalar multiplets in 4, 5 or 6 representations, neutrinos gain masses of a needed value. 4 dimensional

plets are efficient if fundamental scale lies in a range $\sim 10^3$ TeV. 5-plets are motivated for $M_f = $ few $\cdot$ 10 TeV, while for $M_f = $ few TeV 6-plets should be involved. Suggested mechanism can be successfully applied also for low scale SUSY theories.

We also address question of low scale unification which, within low scale theories, has different insight. Due to presence of extra compact dimensions, $SU(3)_c$, $SU(2)_L$, $U(1)_Y$ gauge couplings will have power low running [14] above the scale $1/R$. This give possibility to obtain low scale unification [15]-[17]. For our scenarios, if masses of introduced scalars lie above the GUT scale, then they do not alter renormalization and status of unification would be same as for cases of Refs. [15]-[17]. However, if masses of 4, 5 or 6 plets (depending on a considered scenario) are below the GUT scale, the situation is changed. This open up new possibilities of unification for non SUSY and SUSY scenarios as well. Within non SUSY models introduced $SU(2)_L$ scalars are crucial for successful unification.

2 Suppressed neutrino masses

For generating adequately suppressed neutrino masses, within low scale theories, we will introduce extended $SU(2)_L$ scalars.

Introduce scalar Φ which under $SU(2)_L \times U(1)_Y$ transform as $(4, \ -3)$, where $U(1)_Y$ charge is measured in the units of charge of lepton doublet l. Our studies, of generation suppressed neutrino masses, do not related with existence of extra dimensions. We assume that Φ and all standard model particles are localized on a 3-brane. In order to avoid too large Majorana neutrino masses, somehow we have to forbid $(lh^+)^2/M_f$ type operators (h is SM Higgs doublet). This can be achieved through some symmetries [2]. For simplicity we will assume that lepton number (L) is conserved in the fermion sector (as were done in [11]-[13]) and prescribe to Φ lepton number -2. So, the Yukawa sector possesses $U(1)_L$ symmetry:

$$\mathcal{L}_\nu = \frac{\hat{\lambda}_\nu}{M_f} ll\Phi h + h.c. \ , \tag{1}$$

where $\hat{\lambda}_\nu$ is dimensionless matrix in a family space. As we see, in this case, neutrino masses get additional suppression $\langle h \rangle / M_f$ (in comparison with scenario with scalar triplets [8]). But for low M_f more suppression is needed. Namely, Φ should develop appropriately tiny VEV along its neutral component. This is naturally insured through the scalar potential, with relevant couplings. Most general $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant renormalizable potential is

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[2] In fact, in the Yukawa sector, responsible for generation of charged fermion masses, lepton number is accidentally conserved. Assuming that, this conservation has some fundamental origin, one can extend L conservation also to the appropriate $d = 5$ operators.

$$\mathcal{V}(h, \Phi) = \frac{\lambda_h}{4} \left(h^+ h - m^2\right)^2 + \frac{\lambda_\Phi}{4} \left(\Phi^+ \Phi + M^2\right)^2 +$$

$$\frac{\lambda_1}{2} \left(\Phi^+ \Phi\right) \left(h^+ h\right) + \frac{\lambda_2}{2} \left(\Phi^+ h\right) \left(\Phi h^+\right) - \frac{\lambda}{2} \left(\Phi h^3 + \Phi^+ (h^+)^3\right) , \qquad (2)$$

where all parameters are assumed to be positive. m is Higgs doublet mass of the order of ~ 100 GeV, while M is mass of Φ field. Last term in (2) mildly violates $U(1)_L$. It involves highest power of h and therefore, between last three intersecting quartic terms, will be most suppressed. This is in a good accordance with a so-called naturalness issue [18]. For $\lambda > 0$, system will have global minimum with non zero $\langle \Phi \rangle$. The extremum conditions for (2) will be:

$$\lambda_h (v^2 - m^2) + (\lambda_1 + \lambda_2) V^2 - 3\lambda V v = 0 ,$$
$$\lambda_\Phi (V^2 + M^2) V + (\lambda_1 + \lambda_2) V v^2 - \lambda v^3 = 0 . \qquad (3)$$

For all positive parameters in (2) and for

$$\lambda_\Phi M^2 \gg (\lambda_1 + \lambda_2) m^2 , \qquad (4)$$

one can easily obtain

$$v = m + \mathcal{O}\left(m^3/M^2\right) , \qquad V = \frac{\lambda}{\lambda_\Phi} \left(\frac{v}{M}\right)^2 v + \mathcal{O}\left(m^5/M^4\right) . \qquad (5)$$

Note, that although the mass of Φ is much larger than v, the hierarchy is not destabilized, because Φ's VEV in (5) is tiny and quartic terms in (2) practically do not affect v. For h's potential [first term in (2)] we have used the simplest possible expression. The potential's form for SM doublet is not crucial, because VEV of Φ will have same magnitude as in (5). Important is to achieve desirable electroweak (EW) symmetry breaking. Since this issue is beyond the scope of this paper, we will assume that one of the mechanisms [19], [20], providing EW symmetry breaking (EWSB), for low scale theories, is applied. The natural hierarchy between EW and fundamental scales can be achieved if EWSB occurs dynamically [19], wile SUSY theories (which we consider below) guarantee stability of the scales [condition in (4)].

Using (5) in (1), for neutrino masses we will have

$$\hat{m}_\nu = \hat{\lambda}_\nu \frac{V}{M_f} v \simeq \frac{\lambda \hat{\lambda}_\nu}{\lambda_\Phi} \left(\frac{v}{M}\right)^3 \frac{M}{M_f} v , \qquad (6)$$

and desirable value $\hat{m}_\nu = (1 - 4 \cdot 10^{-2})$ eV is obtained for $M \simeq M_f = (1 - 3) \cdot 10^3$ TeV with $v = 174$ GeV, $\lambda \hat{\lambda}_\nu / \lambda_\Phi \sim 1$. This scale for neutrino masses is natural for atmospheric anomaly [9] if three family neutrinos are either degenerate in mass or hierarchical,

respectively. Smaller scale, relevant for solar neutrinos [10], can be obtained through suppressing the appropriate entries in $\hat{\lambda}_\nu$. Latter can be naturally realized through the flavor symmetries.

If we wish to build scenario with lower fundamental scale, higher $SU(2)_L$ representations must be introduced. Namely, if now Φ is 5-plet of $SU(2)_L$ with $U(1)_Y$ charge -4, then Yukawa couplings, responsible for neutrino masses will be

$$\mathcal{L}_\nu = \frac{\hat{\lambda}_\nu}{M_f^2} ll\Phi h^2 + h.c. \, , \tag{7}$$

and in potential (2) last term will be replaced with $-\frac{\lambda'}{2M_f} \left(\Phi h^4 + \Phi^+ h^{+4} \right)$. For this case it is easy to verify

$$v \simeq m \, , \qquad V \simeq \frac{\lambda'}{\lambda_\Phi} \left(\frac{v}{M} \right)^3 \frac{M}{M_f} v \, . \tag{8}$$

Using (7) and (8), for neutrino masses we will have

$$\hat{m}_\nu = \hat{\lambda}_\nu \frac{V}{M_f^2} v^2 \simeq \frac{\lambda' \hat{\lambda}_\nu}{\lambda_\Phi} \left(\frac{v}{M} \right)^5 \left(\frac{M}{M_f} \right)^3 v \, , \tag{9}$$

which for $\hat{m}_\nu = (1 - 0.1)$ eV, $\lambda'\hat{\lambda}_\nu/\lambda_\Phi \sim 1$ require relatively low scales $M \simeq M_f = (30 - 50)$ TeV.

Fundamental scale can be easily reduced even down to few TeV, if Φ belongs to $(6, \, -5)$ representation of $SU(2)_L \times U(1)_Y$. Then instead the last term in (2) we will have $-\frac{\lambda''}{2M_f^2} \left(\Phi h^5 + \Phi^+ h^{+5} \right)$ and relevant Yukawa couplings will be $\frac{\hat{\lambda}_\nu}{M_f^3} ll\Phi h^3$. By simple analyses one can easily obtain that in this case

$$\hat{m}_\nu \simeq \frac{\lambda'' \hat{\lambda}_\nu}{\lambda_\Phi} \left(\frac{v}{M} \right)^7 \left(\frac{M}{M_f} \right)^5 v \, , \tag{10}$$

and $(1 - 0.1)$ eV neutrino masses (for $\lambda''\hat{\lambda}_\nu/\lambda_\Phi \sim 1$) is generated for $M \simeq M_f = (7 - 10)$ TeV.

These scenarios can be successfully extended to the low scale supersymmetric theories. In SUSY versions, together with chiral superfield Φ (which denote 4, 5 or 6-plets) must be introduced conjugate $\overline{\Phi}$ supermultiplet. The relevant superpotential will be

$$W_\Phi = M\overline{\Phi}\Phi - \frac{1}{M_f^{1+n}} \left(\lambda_{\Phi d}\Phi h_d^{3+n} + \lambda_{\Phi u}\overline{\Phi} h_u^{3+n} \right) \, , \tag{11}$$

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where $n = 0, 1, 2$ for scenarios with $\Phi + \overline{\Phi}$ in 4, 5 and 6 representations of $SU(2)_L$ respectively. h_u, h_d denote doublet-untidoublet pair of MSSM and $\lambda_{\Phi d}$, $\lambda_{\Phi u}$ are positive dimensionless coupling constants. Yukawa superpotential, responsible for neutrino masses, will be

$$W_\nu = \frac{\hat{\lambda}_\nu}{M_f^{n+1}} l l \Phi h_d^{n+1} . \tag{12}$$

In unbroken SUSY and EW symmetry limit $\langle h_u \rangle = \langle h_d \rangle = 0$, and from (11) follows also $\langle \Phi \rangle = \langle \overline{\Phi} \rangle = 0$. After that SUSY and EW symmetry breaking take place[3], non zero $\langle h_u \rangle$, $\langle h_d \rangle$ are generated and from (11) one can easily verify $\langle \Phi \rangle \simeq \lambda_{\Phi u} \langle h_u \rangle^{n+3} / (M M_f^{n+1})$. Using this and also (12), for neutrino masses we will get

$$\hat{m}_\nu = \hat{\lambda}_\nu \lambda_{\Phi u} \left(\frac{v}{M_f} \right)^{2n+2} \frac{v^2}{M} \sin^{n+3} \beta \cos^{n+1} \beta , \tag{13}$$

where we have used $\langle h_u \rangle = v \sin \beta$, $\langle h_d \rangle = v \cos \beta$. As we see, within SUSY scenarios expressions for neutrino masses are slightly modified [compare with (6), (9), (10)]. However, needed suppressions are still guaranteed. In particular, for $\hat{\lambda}_\nu \lambda_{\Phi u} \lesssim 1$, $v = 174$ GeV and $\tan \beta \simeq 1$, neutrino masses $m_\nu \lesssim (1 - 0.1)$ eV are obtained within various scenarios:

$$M \simeq M_f = \begin{cases} (0.6 - 1.3) \cdot 10^3 \text{ TeV}; & n = 0, \text{ case with } 4 - \text{plets} \\ (20 - 30) \text{ TeV}; & n = 1, \text{ case with } 5 - \text{plets} \\ (4.7 - 6.5) \text{ TeV}; & n = 2, \text{ case with } 6 - \text{plets} \end{cases} . \tag{14}$$

Larger values of $\tan \beta$ would give stronger suppression of neutrino masses in (13), giving possibility to reduce mass scales in (14) by few factors.

Obtaining ranges for scales, in (6), (9), (10) and (14), we have assumed $M \simeq M_f$ (and $\tan \beta \simeq 1$ for SUSY cases). Obviously, it is possible to have M_f by few factors larger than the value of M. This will slightly modify the ranges for mass scales. Important point is, that mechanisms which we have suggested here, provide adequate suppressions of neutrino masses and this suppressions occur through proper choice of scalar Φ in appropriate $SU(2)_L \times U(1)_Y$ representation.

3 Gauge coupling unification

If extra spacelike dimensions exist, it is possible to obtain the low scale unification of gauge coupling constants. This can take place if scale $\mu_0 = 1/R < M_G$. Above the scale

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[3]Still, it is assumed that one of the mechanisms, for SUSY and EW symmetry breaking, is applied (see [21] and [19], [20] respectively).

μ_0 the heavy Kaluza-Klein (KK) states enter into the game and gauge coupling runnings become power low. The solution of one loop RGEs have forms [14]-[17]:

$$\alpha_G^{-1} = \alpha_a^{-1} - \frac{b_a}{2\pi} \ln \frac{M_G}{M_Z} + \Delta_a \ , \tag{15}$$

where $\alpha_{1,2,3}$ denote gauge couplings (on scale M_Z) of $U(1)$, $SU(2)_L$ and $SU(3)_c$ respectively, b_a is standard b-factors (depending which theory we are studying - non-SUSY or SUSY). In general, in Δ_a could contribute two type of terms

$$\Delta_a = \Delta_a^0 + \Delta_a^{KK} \ , \tag{16}$$

where Δ_a^0 denote contribution of some additional states with masses M_i below the GUT scale, and have logarithmic energy dependence

$$\Delta_a^0 = -\frac{\tilde{b}_a^i}{2\pi} \ln \frac{M_G}{M_i} \ . \tag{17}$$

Δ_a^{KK} express contribution of KK states and have power low energy dependence [14]

$$\Delta_a^{KK} = -\frac{\hat{b}_a^i}{2\pi} P_\delta^{(\mu_i)} \ , \qquad P_\delta^{(\mu_i)} = \frac{X_\delta}{\delta} \left[\left(\frac{M_G}{\mu_i} \right)^\delta - 1 \right] - \ln \frac{M_G}{\mu_i} \ , \tag{18}$$

where $X_\delta = \pi^{\delta/2}/\Gamma(1 + \delta/2)$, $\mu_i^2 = M_i^2 + \mu_0^2 = M_i^2 + 1/R^2$ (for SM and MSSM states $M_i = 0$). For simplicity we have assumed that all δ compact extra spacelike dimensions have equal radius. From (15), excluding α_G and $\ln(M_G/M_Z)$, for strong coupling we find:

$$\alpha_s^{-1} = \frac{b_1 - b_3}{b_1 - b_2}(\alpha_2^{-1} + \Delta_2) - \frac{b_2 - b_3}{b_1 - b_2}(\alpha_1^{-1} + \Delta_1) - \Delta_3 \ , \tag{19}$$

and for given values of Δ_i we can estimate the value of $\alpha_s(M_Z)$. Also, through (15) one can calculate the value of GUT scale

$$\ln \frac{M_G}{M_Z} = \frac{2\pi}{b_1 - b_2} \left(\alpha_1^{-1} - \alpha_2^{-1} + \Delta_1 - \Delta_2 \right) \ , \tag{20}$$

and finally, the value of unified gauge coupling constant

$$\alpha_G^{-1} = \frac{1}{b_1 - b_2} \left[b_1(\alpha_2^{-1} + \Delta_2) - b_2(\alpha_1^{-1} + \Delta_1) \right] \ . \tag{21}$$

Through analyzes one has to make sure that α_G remains in a perturbative regime.

In case with $\tilde{b}_a^i = 0$ (or $M_i \gtrsim M_G$) and $\delta = 0$, $\Delta_a^0 = \Delta_a^{KK} = 0$ and the status of unification is unchanged. In case, when there are no additional states (e.g. $\Delta_a^0 = 0$) and

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we have only KK excitations (either of SM or MSSM states), the unification picture is not altered if $\hat{b}_a$ factors satisfy condition:

$$\frac{\hat{b}_a - \hat{b}_b}{b_a - b_b} = const. \quad (\text{for } a \neq b) . \tag{22}$$

For SM $b_a = (41/10, -19/6, -7)$, and case with $\Delta_a = 0$ predicts $\alpha_s = 0.071$, which is unacceptable [22]. As was pointed out in [15], the existence of extra dimensions open up possibilities for improving this situation. Namely, if conditions in (22) are mildly violated then one can attempt to get successful low scale unification. This take place by introducing three real $SU(2)_L$ adjoint scalars only with KK excitations and no zero mode wave functions[4].

In MSSM, in one loop approximation, we have successful picture of unification and for its preserving conditions in (22) must be satisfied (at least in a high accuracy). This can be reached by introducing additional pairs of vectorlike chiral superfields [16, 17].

For our scenarios, either with 4, 5 or 6-plets, all above mentioned cases of unification will be achieved if their masses M are not below the GUT scale. Successful unification will take place if the ideas of [15] and [16, 17] will be applied for non-SUSY and SUSY cases respectively. However, cases with $M < M_G$ will give different results and we would like to study these examples here.

Let us start with non-SUSY case with 4-plets. As it will turn out, these states are crucial for unification. For n_Φ 4-plets with masses $M < M_G$ we have

$$\tilde{b}_a = (\frac{9}{5}, \ \frac{5}{3}, \ 0)n_\Phi , \qquad \hat{b}_a = \hat{b}_a^{\text{SM}} + (\frac{9}{5}, \ \frac{5}{3}, \ 0)n_\Phi , \tag{23}$$

where $\hat{b}_a^{\text{SM}} = (1/10, -41/6, -21/2) + (8/3, 8/3, 8/3)\eta$ (η is number of chiral families with KK excitations).

Ignoring Δ_a^{KK}, we will have $\alpha_s^{-1} = (\alpha_s^{-1})_{\text{SM}}^0 - \frac{87}{109\pi}n_\Phi \ln \frac{M_G}{M}$, and for $M_G/M \simeq 40$, $n_\Phi = 6$, we obtain $\alpha_s = 0.119$. But without KK excitations there is no power low running and according to (20) no low scale unification is obtained[5]. As it turns out, unification take place for $\mu_0 < M < M_G$. It means that we have to include KK excitations of $\Phi(4)$ states starting only from scale M, while KK states of SM particles enter into the game from μ_0 scale. For α_s we get:

$$\alpha_s^{-1} = (\alpha_s^{-1})_{\text{SM}}^0 - \frac{87}{109\pi}n_\Phi \ln \frac{M_G}{M} - \frac{87}{109\pi}n_\Phi P_\delta^{(M)} - \frac{1}{218\pi}P_\delta^{(\mu_0)} , \tag{24}$$

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[4]This is fully consistent with an orbifold compactification scenarios. We do not go through latter issue here and refer the reader to [15], where detailed discussions are presented.

[5]See however [23], where examples of low scale unification, without extra dimensions, were presented.

where $P_\delta^{(M)}$ and $P_\delta^{(\mu_0)}$ denote functions presented in (18), calculated for appropriate scales. In (24) the n_Φ and ratios M_G/M, M_G/μ_0 must be chosen in such a way as to get desirable value for α_s. At the same time from (20) for M_G we should get reasonable value (not too larger than M) and also α_G in (21) must be in a perturbative regime. Also, the values of M, M_f should be such that neutrino masses must be properly suppressed in (6). We will assume that $M_G \simeq M_f$ and require $m_\nu \lesssim 1$ eV. Taking into account all this, from (24), (20), (21) it is easy to see that successful unification with $\alpha_s \simeq 0.119$ is obtained for $n_\Phi = 2$ and various values of extra dimensions and mass scales:

$$\left(\delta, \; \frac{M_G}{\mu_0}, \; \frac{M_G}{M}, \; M_G\right) = \left(1, \quad 9.78, \quad 6.51, \quad 10^{3.51} \text{ TeV}\right),$$

$$\left(2, \quad 3.45, \quad 2.83, \quad 10^{3.26} \text{ TeV}\right), \; \left(3, \quad 2.36, \quad 2.071, \quad 10^{3.18} \text{ TeV}\right), \cdots \quad (25)$$

The values of α_s and M_G are η independent, while α_G in (21) depends on η. In this case for $0 \overset{\leq}{} \eta \overset{\leq}{} 3$ the α_G remains in a perturbative regime $2 \cdot 10^{-2} \alpha_G < 4 \cdot 10^{-2}$. Result of numerical calculation for $\delta = 1$, $\eta = 0$ is presented on Fig. 1, (a).

Similar discussions and analyses can be done for non-SUSY scenario with 5-plets. In this case

$$\alpha_s^{-1} = (\alpha_s^{-1})_{\text{SM}}^0 - \frac{325}{218\pi} n_\Phi \ln \frac{M_G}{M} - \frac{325}{218\pi} n_\Phi P_\delta^{(M)} - \frac{1}{218\pi} P_\delta^{(\mu_0)}, \quad (26)$$

and $\alpha_s \simeq 0.119$ is obtained for $n_\Phi = 1$, with

$$\left(\delta, \; \frac{M_G}{\mu_0}, \; \frac{M_G}{M}, \; M_G\right) = \left(1, \quad 11.55, \quad 6.9, \quad 10^{1.82} \text{ TeV}\right),$$

$$\left(2, \quad 3.735, \quad 2.92, \quad 10^{1.67} \text{ TeV}\right), \; \left(3, \quad 2.486, \quad 2.12, \quad 10^{1.61} \text{ TeV}\right), \cdots \quad (27)$$

Also in this case for $0 \overset{\leq}{} \eta \overset{\leq}{} 3$, the α_G remains in perturbative regime $(\lesssim 4 \cdot 10^{-2})$. Unification picture for this scenario, for $\delta = 1$, $\eta = 0$, is illustrated on Fig. 1, (b).

As far, the scenario with 6-plets is concerned, unification near few TeV energies is obtained (with $\alpha_s \simeq 0.119$) for $n_\Phi = 1$ and

$$\left(\delta, \; \frac{M_G}{\mu_0}, \; \frac{M_G}{M}, \; M_G\right) = (1, \quad 12.25, \quad 4.55, \quad 10.7 \text{ TeV}),$$

$$(2, \quad 3.837, \quad 2.35, \quad 8.69 \text{ TeV}), \; (3, \quad 2.486, \quad 2.12, \quad 8.11 \text{ TeV}), \cdots \quad (28)$$

Fig. 1, (c) corresponds to this case with $\delta = 1$, $\eta = 0$.

Let us now turn to the SUSY cases. In higher dimensional theories all introduced states must be embedded in $N = 2$ supermultiplets. We will assume that MSSM doublet-untidoublet form one $N = 2$ supermultiplet (h_u, h_d). And also each pair of $\Phi + \overline{\Phi}$ form

one $N = 2$ supermultiplet $(\Phi, \overline{\Phi})$. Since one loop value of α_s in MSSM is 0.117, the new contributions within our scenarios should not be large. As it turn out, for successful unification, either with 4, 5 or 6 supermultiplets, we have to introduce one additional state in the $SU(3)_c$ adjoint representation only with KK excitations and without zero mode wave function. With this, for n_Φ pairs of $4 + \bar{4}$ we have:

$$\alpha_s^{-1} = (\alpha_s^{-1})^0_{\text{MSSM}} - \frac{33}{7\pi}n_\Phi \ln \frac{M_G}{M} - \frac{33}{7\pi}n_\Phi P_\delta^{(M)} + \frac{39}{14\pi}P_\delta^{(\mu_0)} \; , \tag{29}$$

and its desirable value 0.119 and successful unification is obtained for $n_\Phi = 1$ and

$$\left(\delta \; , \; \frac{M_G}{\mu_0} \; , \; \frac{M_G}{M} \; , \; M_G\right) = \left(1, \quad 18.57, \quad 10.56, \quad 10^{3.13} \text{ TeV}\right) \; ,$$

$$\left(2, \quad 4.78, \quad 3.657, \quad 10^{2.96} \text{ TeV}\right) \; , \; \left(3, \quad 2.937, \quad 2.465, \quad 10^{2.92} \text{ TeV}\right) \; , \cdots \tag{30}$$

In this case η must be zero, since its higher values drive α_a couplings in non perturbative regime until they reach unification point. For this scenario unification picture for $\delta = 1$, $\eta = 0$ is plotted on Fig. 1, (d).

As far the case with 5 dimensional supermultiplets are concerned, also for this scenario successful pictures of unification will be obtained for $n_\Phi = 1$, with presence of same $SU(3)_c$ adjoint (as in the case above). Namely, $\alpha_s \simeq 0.119$ is obtained for

$$\left(\delta \; , \; \frac{M_G}{\mu_0} \; , \; \frac{M_G}{M} \; , \; M_G\right) = \left(1, \quad 18.02, \quad 6.09, \quad 10^{1.46} \text{ TeV}\right) \; ,$$

$$\left(2, \quad 4.683, \quad 2.742, \quad 10^{1.39} \text{ TeV}\right) \; , \; \left(3, \quad 2.895, \quad 2.029, \quad 10^{1.36} \text{ TeV}\right) \; , \cdots \tag{31}$$

Also now only $\eta = 0$ case is allowed. Unification picture for $\delta = 1$ is plotted on Fig. 1, (e).

For SUSY scenario with 6-plets, successful unification take place for $n_\Phi = 1$ and

$$\left(\delta \; , \; \frac{M_G}{\mu_0} \; , \; \frac{M_G}{M} \; , \; M_G\right) = (1, \quad 16.84, \quad 3.9, \quad 5.74 \text{ TeV}) \; ,$$

$$(2, \quad 4.515, \quad 2.169, \quad 5.25 \text{ TeV}) \; , \; (3, \quad 2.823, \quad 1.729, \quad 5.12 \text{ TeV}) \; , \cdots \tag{32}$$

Also, in this scenario only $\eta = 0$ is allowed and unification picture for $\delta = 1$ is plotted on Fig. 1, (f).

As we have seen, successful unifications for non-SUSY and SUSY scenarios can be obtained even for Φ-plet masses M below the GUT scale. For all this cases unification take place not too far from the scale M (for illustrations see Fig. 1).

Through analyses, for neutrino masses in (6), (9), (10) and (13), we have taken $M_f \simeq M_G$. However, it is possible to have unification scale, by few factors and even more, below

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the M_f. This would reduce scales μ_0 and M, making scenarios easily testable on a future colliders.

4 Conclusions

In this paper we have suggested mechanism for natural suppressing neutrino masses, within theories of low scale quantum gravity. Crucial role is played by Φ scalars in different representations of $SU(2)_L$. Selection of Φ is dictated from the value of fundamental scale. Different scenarios were considered, in which neutrino masses are suppressed in the needed level. Further studies, of low scale theories with those Φ states, would be an attempt to accommodate atmospheric and solar neutrino anomalies [9, 10]. For this purpose, one can also introduce flavor symmetries and build different neutrino oscillation scenarios in a spirit of [24]. The flavor symmetries within low scale theories could play crucial role for suppression of FCNC together with natural understanding of hierarchies of the charged fermion masses and CKM mixings [25]. Particular interest deserve scenarios with fundamental scales (and consequently masses of Φ states) close to TeV range (cases with 5 and 6-plets),being testable in a collider experiments of a nearest future.

Introduced Φ states (together with appropriate KK excitations) are also crucial for non-SUSY low scale unification, while for SUSY scenarios $\Phi + \overline{\Phi}$ supermultiplets open up new possibilities for successful unification. In considered examples, unification points are close to the fundamental scale (few or multi TeV) and building realistic models, one have to take care for nucleon stability. For latter, one of the mechanisms suggested in [15, 26] could be efficient. Detailed investigations and studies of these and related issues will be presented elsewhere.

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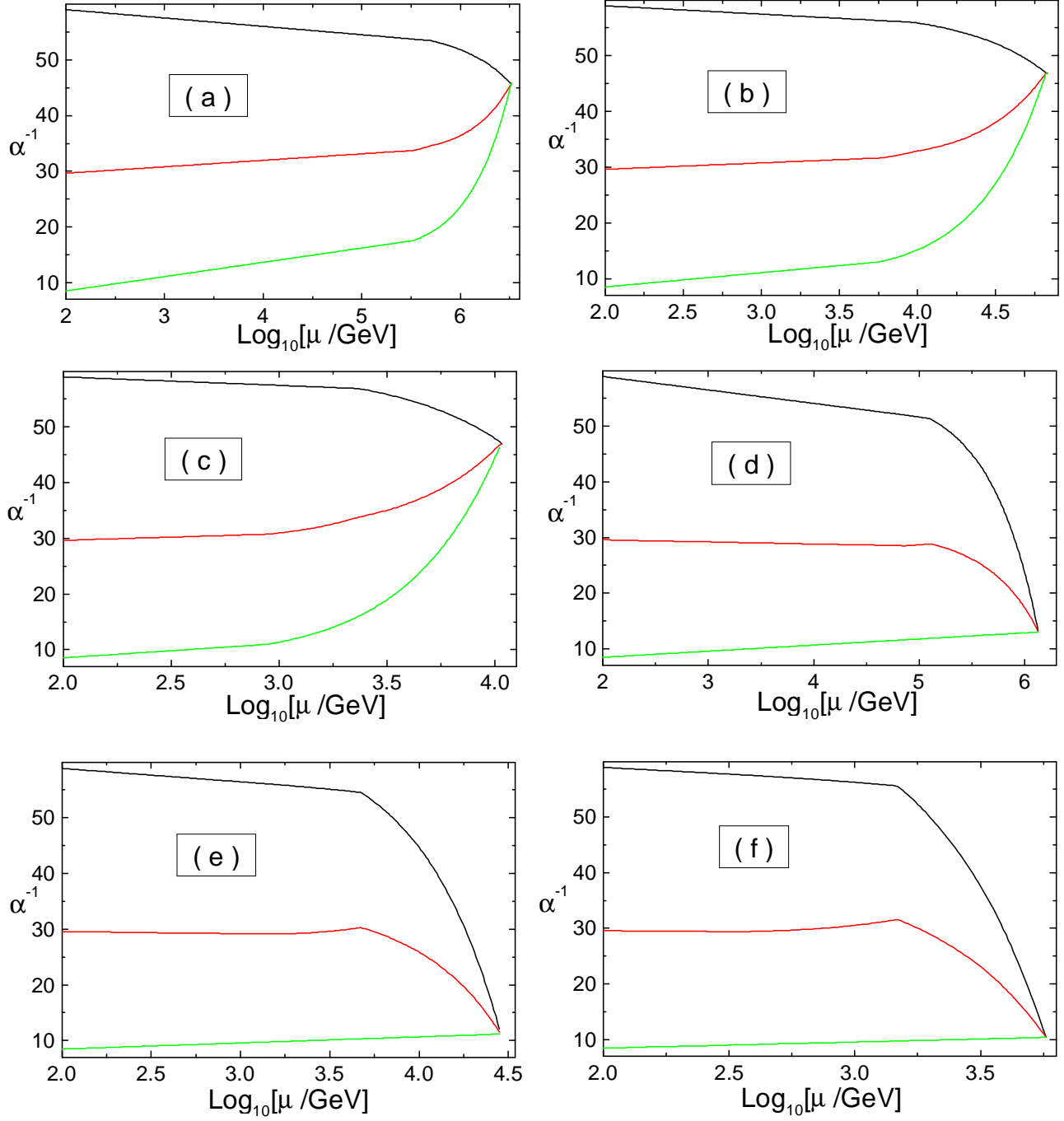
Figure 1: Unification pictures for $\alpha_s(M_Z) \simeq 0.119$ and $\delta = 1$, $\eta = 0$; (a), (b), (c) non-SUSY cases with two scalar 4, one 5 and one 6 plets respectively; (d), (e), (f) SUSY cases with $\tan \beta \simeq 1$ and one pair of chiral 4, 5 and 6 supermultiplets respectively.

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